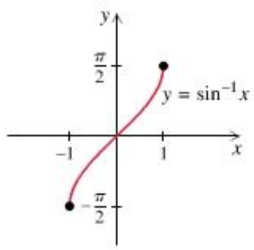
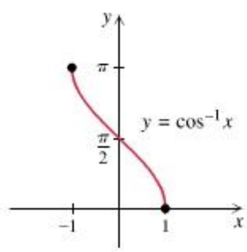
The Properties of the Inverse Secant and Cosecant Functions

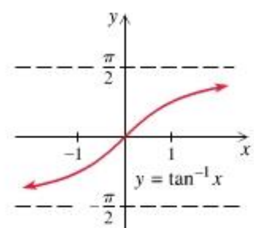
In order for a function to have an inverse, the function must be one-to-one. In other words, it must pass the horizontal line test. Since the trigonometric functions are periodic, we must restrict the domain so that they will pass the horizontal line test. Therefore, the inverse functions will have a restricted range.



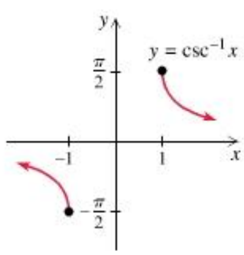
Domain $[-1, 1]$
Range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



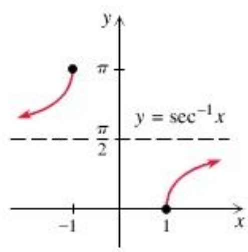
Domain $[-1, 1]$
Range $[0, \pi]$



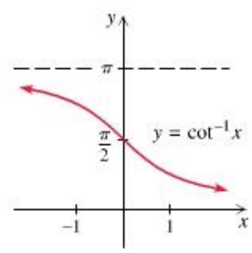
Domain $(-\infty, \infty)$
Range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Domain $(-\infty, -1] \cup [1, \infty)$
Range $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$



Domain $(-\infty, -1] \cup [1, \infty)$
Range $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



Domain $(-\infty, \infty)$
Range $(0, \pi)$

Inverses of the Reciprocal Trigonometric Functions

$\csc^{-1}(x)$ is the angle in $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ whose cosecant is *x*.

$\sec^{-1}(x)$ is the angle in $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ whose secant is *x*.

Identities

$$\csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$$

$$\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$$

Example: Find the exact value of each expression without using a table or calculator.

a) $\operatorname{arcsec}(-2)$ b) $\csc^{-1}\left(\sqrt{2}\right)$ c) $\sec^{-1}\left(\sec\left(\frac{5\pi}{4}\right)\right)$ d) $\cot\left(arc\csc(-3)\right)$

Example: Find the approximate value of each expression rounded to 4 decimal places.

a) $\operatorname{arccsc}(-1.4713)$ b) $\sec^{-1}(4.328)$

Example: Rewrite the following as algebraic expressions of x and state the domain on which the equivalence is valid.

a) $\tan\left(arc\sec(x)\right)$ b) $\cos\left(\csc^{-1}(4x)\right)$